Derivatives of Logarithmic Functions

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A) $y = \log_5(x^3)$

$\frac{dy}{dx} = \frac{1}{x^3 \cdot \ln 5} \cdot 3x^2$

$= \frac{3x^2}{x^3 \ln 5} = \boxed{\frac{3}{x \ln 5}}$

B) $y = \log_6 \sqrt[3]{x}$ $\quad y = \log_6(x^{1/3})$

$y' = \frac{1}{x^{1/3} \cdot \ln 6} \cdot \frac{1}{3} x^{-2/3}$

$y' = \frac{1}{3 x^{2/3} x^{1/3} \ln 6} = \boxed{\frac{1}{3x \ln 6}}$

$y = \log_5(4x^{-1})$

C) $y = \log_5\left(\frac{4}{x}\right)$

$y' = \frac{1}{(4x^{-1}) \ln 5} \cdot -4x^{-2} = \frac{-4x}{4x^2 \ln 5}$

$= \boxed{\frac{-1}{x \ln 5}}$

D) $y = \frac{5}{\log_7(x^2)}$

$y = 5[\log_7(x^2)]^{-1}$

$y' = -5[\log_7(x^2)]^{-2} \cdot \frac{1}{x^2 \ln 7} \cdot 2x$

(D) $\frac{-10}{x \ln 7 [\log_7(x^2)]^2}$

E) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x \ln e} \cdot (1)$

$= \frac{1}{x}$

F) $y = \ln(x^4)$

$y' = \frac{1}{x^4 \ln e} \cdot 4x^3$

$\boxed{y' = \frac{4}{x}}$

$y = \ln^4 x$ ←

G) $y = (\ln x)^4$

$y = (\ln x)^4$

$y' = 4(\ln x)^3 \cdot \frac{1}{x}$

H) $y = \ln\left(\frac{5}{x}\right)$ → $y = \ln 5x^{-1}$

$y = \ln 5 - \ln x$

$y' = 0 - \frac{1}{x} = \boxed{-\frac{1}{x}}$

I) $y = x^3 \ln(x^2) - \ln(\ln(\arcsin x))$

$y' = x^3\left[\frac{1}{x^2 \ln e} \cdot 2x\right] + \ln(x^2)(3x^2) - \left[\frac{1}{\ln(\arcsin x) \cdot \ln e} \cdot \frac{1}{\arcsin x \cdot \ln e} \cdot \frac{1}{\sqrt{1-x^2}}\right]$

$y' = \frac{2x^4}{x^2} + 3x^2 \ln(x^2)$

$y = 2x^2 + 3x^2 \ln(x^2) - \frac{1}{\sqrt{1-x^2} \ln(\arcsin x) \cdot \arcsin x}$